Recurrence Relation:-

It is an equation that defines a equation recursively.

Substitution Method:-

(i) Forward Substitution:- NOT used Frequency

uses the initial condition in the initial term and value for the next term is generated.

e.g $T(n) = T(n-1) + n, \quad T(0) = 0. \} \; n > 0$

↳ Inital Condn.

$n = 1$, then

using inital Condn.

$T(1) = T(0) + 1$

$T(1) = 1$ —— (1)

$n = 2$ then

$T(2) = T(1) + 2$

$T(2) = 1 + 2 = 3$ —— (2)

$n = 3$ then

$T(3) = T(2) + 3$

$= 3 + 3 = 6$ —— (3)

$T(n) = n(n+1)/2 \}$ Guessed from eqn (1) (2) & (3)

$=$

$T(n) = \frac{n^2}{2} + \frac{n}{2}$

$$\boxed{T(n) = O(n^2)}$$

Backward Substitution:- In this backward values are substituted recursively.

Eg:- $T(n) = T(n-1)+n$ with $T(0)=0$ —(i)

$T(n-1) = T(n-1-1)+(n-1)$ } Replace n by n-1 in (i)

$T(n-1) = T(n-2)+(n-1)$ —(ii)

$T(n-2) = T(n-2-1)+(n-2)$ { replace n by n-2

$= T(n-3)+(n-2)$ —(iii)

$= T(n-k)+(n-k+1)+(n-k+2)+\cdots n$

if $k=n$, then

$T(n) = T(0)+1+2+\cdots n$

$T(n) = \underbrace{0+1+2+\cdots+n}_{\text{sum of n natural no}}$

$T(n) = n(n+1)/2$

$T(n) = \frac{n^2}{2}+\frac{n}{2}$

$\boxed{T(n) = O(n^2)}$